Retake Test 1 Computational Methods of Science, February 3, 2023

Duration: 1 hour.
In front of the questions, one finds the points. The sum of the points plus 1 gives the end mark for this test. Criteria used for the grading are: demonstration of understanding, logical reasoning, correct use of terminology, correctness of results.

Consider on (0,1) the differential equation

$$-\exp(x)\frac{d^2u}{dx^2} - \exp(x)\frac{du}{dx} = \tan(x)$$

with boundary conditions $\frac{du}{dx}(1) + 10u(1) = -5$ and $u(0) = 1$.

1. [3.5] Derive the weak (Galerkin) form and the associated function space of this problem. Give also the bilinear and linear form, where in the bilinear form only first-order derivatives appear.

2. [1.5] Choose the remaining freedom in the bilinear form such that it becomes nonnegative. If you were not able to solve 1, then use $a(v, u) = (dv/dx - v, \exp(-x)(du/dx - u)) + (2 - \alpha)v(1)du/dx(1) + 6\alpha v(1)u(1)$.

3. [0.5] Assume that the bilinear form derived in 1 and 2 is coercive. Is there an associated minimization problem? Explain your answer.

4. [3.5] Consider the space V_h of piecewise linear interpolation polynomials with interpolation points $x_i = ih$ with $h = 1/n$ and $v_h(0) = 0$ for all $v_h \in V_h$. Give the linear basis $\{\phi_1, \cdots, \phi_n\}$ for V_h. Using these linear basis functions, give, in integral form, the diagonal elements A_{ii} for $i = 1, \cdots, n$ of the matrix A which occurs when approximating the solution on the space V_h, i.e. $u(x) \approx \sum_{i=1}^{n} c_i\phi_i(x)$. Develop your answer such that the next step woould be actually computing the occurring integrals. If you were not able to solve question 1, you may use the bilinear form given in question 2.